Degrees of Specialization among Workers of Differing Ability *

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Abstract. It is not obvious whether a firm's more talented workers should be more specialized or not, and in fact, the relationship between ability and specialization seems to differ across industries. To understand these differences, I incorporate "general" skill or ability into a Becker and Murphy (1992) model of specialization and show that when general skill is substitutable (complementary) for specialized knowledge in production, then efficiency requires that workers with greater levels of general skill should specialize less (more).

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I. Introduction

Corporate managers with higher general human capital usually manage firms with greater spans of control;[1] on the other hand, better-qualified physicians and chefs generally specialize to a high degree. Differences like these suggest differences in the production function for different worker outputs. This paper shows that only when general human capital (or ability) is significantly substitutable for specialized skill in production should the more able workers tend to generalize. In such a case, specialized skill in a particular task is marginally less useful to the abler workers, allowing them to substitute their high general human capital over a larger variety of tasks.

Herein, I develop a model of specialization, extending the strain of literature on "endogenous comparative advantage" models derived from Adam Smith (1776), Rosen (1983) and Becker and Murphy (1992).[2] Unlike "exogenous" comparative advantage models of specialization[3], where workers specialize due to heterogeneity in specific abilities, endogenous comparative advantage models consider workers who are ex ante identical, but who nevertheless choose to specialize because of increasing returns to scale.[4]

I extend this literature by explicitly endogenizing a worker's choice of the degree of specialization through a model in which workers contrast the increased productivity from specialization with some costs to specialization (for instance, increased job search time, risk, pure boredom, or even Marxian "alienation"). Moreover, in order to explain the observed heterogeneity in the degree of specialization between workers, I modify the usual "ex ante identical" assumption in endogenous comparative advantage models to include differences in

[1] See Gulamhusein (2005).
[2] Becker(1991), Yang and Borland(1991), Tamura (1992), and Garicano and Hubbard (2003), among others, have used endogenous comparative advantage models to explore a variety of important social phenomena.
[3] Also known as "Ricardian" models. See Rosen (1978).
[4] Yang and Ng (1998) provide a survey of the differences between exogenous and endogenous models.

general ability across workers[5]. I show that the choice of specialization is a function of the substitutability or complementarity between ability and specialized skill[6]. Thus, given different production processes, different industries may evince different patterns in the relationship between ability and specialization.

II. The Model

Workers allocate their production time across different tasks. Each task yields an independent task-specific output. In order to produce a particular task-specific output, a worker must devote time to investing in the specific human capital associated with that task, as well as devote time to actually performing the task. Workers thus choose over three variables: the number of different types of specific human capital they acquire; the total amount of time they devote to each task; and the division of their time on each task between investing and production.

Let Z denote the set of tasks. A worker's productivity per unit time on any task $z \in Z$, is denoted E^z, and is a function of his exogenous ability or general human capital, denoted h, and the amount of time he has spent investing in the specialized human capital associated with task z, denoted t_s^z:

[1] $$E^z = E(h, t_s^z)$$

By assumption, all tasks are equally difficult, so the function E need not be superscripted with z. E is assumed to be increasing and concave in both its arguments. A worker's total output in a task z, Y^z, may then be denoted

[2] $$Y^z = E^z t_w^z$$

[5] MacDonald and Markusen(1985) similarly consider a model with both comparative and absolute advantages in some tasks, though in a different framework.

[6] This result echoes the findings of span-of-control theory, which finds correlations between entrepreneurial ability and firm size (see Rosen, 1982, and Garicano, 1998)

where t_w^z is the time spent producing task *z*.[7] The total time devoted to the task is then

$t^z = t_s^z + t_w^z$.

One may solve the worker's problem in two stages. First, the worker chooses t_s^z and t_w^z, taking the total amount of time allocated to task *z*, t^z, as given.

For each *z*, then, the problem of dividing t^z between t_s^z and t_w^z is:

[3] $$\max_{t_w^z, t_s^z} E(h, t_s^z) t_w^z \quad \text{subject to } t^z = t_s^z + t_w^z$$

From the first order condition of this problem, it can be seen that

[4] $$t_w^z = \frac{E(h, t^z - t_w^z)}{E_t(h, t^z - t_w^z)}$$

where subscripts on the function *E* indicate the derivative with respect to the subscripted argument. Note that, according to equation [4], t_w^z may be written as a function of t^z, $t_w^z = f(t^z)$. Taking the implicit derivative (and suppressing the arguments of E for readability), we find:

[5] $$f_t(t^z) = \frac{E_t^2 - EE_{tt}}{2E_t^2 - EE_{tt}} \in [0,1]$$

Substituting equation [4] into equation [2], task z's output can be written as a function of t^z alone:

[6] $$Y^z(h, t^z) = E(h, t^z - f(t^z)) f(t^z)$$

Proposition 1. There are increasing returns to specialization; that is, $Y_{tt}^z > 0$.

Proof. Dropping the superscript z,

[7] All hours of work have the same productivity E^z. This assumption can be relaxed to allow marginal productivity to decrease in t_w^z without changing the results. Baumgartner (1988) develops a model in which workers have some market power, and thus face declining demand curves for task-specific outputs.

$$Y_t = E_t f(1-f_t) + Ef_t > 0$$

due to the bounds on f_t given in equation [6]. Then note that

$$\begin{aligned} Y_{tt} &= 2E_t f_t (1-f_t) + E_{tt} f(1-f_t)^2 + (E - E_t f) f_{tt} \\ &= (1-f_t)\big(2E_t f_t + E_{tt} f(1-f_t)\big) + (E - E_t f) f_{tt} \end{aligned}$$

Now, substituting in equations [4] and [5], the second term disappears, and the first term is then

$$\begin{aligned} Y_{tt} &= (1-f_t)\left(2E_t\left[\frac{E_t^2 - EE_{tt}}{2E_t^2 - EE_{tt}}\right] + E_{tt}\left[\frac{E}{E_t}\right]\left[\frac{E_t^2}{2E_t^2 - EE_{tt}}\right]\right) \\ &= (1-f_t)\left(\frac{2E_t^3}{2E_t^2 - EE_{tt}}\right) \end{aligned}$$

Since $f_t \in [0,1]$ by equation [6], and E is positive, increasing, and concave, this expression is positive. **QED**.

The increasing returns property demonstrated in Proposition 1 is due to the fact that the stock of specialized knowledge is not "used up" through production of the specialized output, and hence constitutes a fixed cost to production, a feature this model generalizes from Becker and Murphy (1992).

Consider now the second stage of a worker's choice, that of determining the optimal t^z (= $t_s^z + t_w^z$) for each *z*. Assume that workers each have the same total amount of time *T* outside of leisure, and thus that $\sum_Z t^z = T$. If there were no costs to specialization, Proposition 1 implies that workers would specialize completely, i.e., $t^z = T$ for some arbitrary z. The fact that generalists exist implies some costs to specialization.

One cost is simply the boredom or social alienation entailed in specialization. In addition, Murphy (1986) and Carrington (1990) identify uncertainty about the future prices of specialized outputs as an incentive to generalize, Malamud (2004) suggests that specialized workers have lower match quality with their chosen occupation, and Kim (1989) theorizes that search costs of employment increase with specialization.[8]

To incorporate these considerations, the cost of specialization is assumed to be continuously decreasing in the relative equality of the time allocations (t^z) to the chosen tasks, and decreasing in the (discrete) number of tasks chosen:[9]

$$[7] \qquad C = C(\sum_{z} (t^z)^2, \sum_{z} \lambda(t^z > 0)) = C(HHI, N)$$

where λ is the indicator function. Note that the first argument is the Herfindahl Index and the second argument is the number of tasks the worker performs in positive quantity.

While it may seem at first unintuitive that both the arguments included in the cost function specified above are necessary, it can be readily checked that alternative specifications without both arguments lead to corner solutions of either complete specialization or complete generalization.[10]

Then the worker's maximization problem associated with the "second stage" – setting t^z for each z – is given by:

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[8] In a more general model with complementary task-outputs, costs relating to coordination between workers may rise with increased specialization as well. See Chari and Jones (2000) or Becker and Murphy (1992).

[9] This reflects an assumption that the costs of specialization decrease with generality, but that there is a discrete cost of knowing absolutely *nothing* about any particular subject. For instance, knowing just the names of car parts can allow one to appear informed when conversing with auto mechanics and significantly reduce the likelihood of fraud.

[10] If specialization costs are simply a decreasing function of N, workers will specialize "almost" completely, i.e., set $t^z \approx T$ for one z, and $t^z = \varepsilon$ for all other z. Thus, they would minimize costs while retaining the returns from specialization. Alternatively, if the costs of specialization were simply decreasing in HHI alone, workers would either specialize completely or generalize completely, depending on the relative convexity of the cost function and Y. Such functional forms seem inadequate to explain differential specialization choices among workers. Specialization costs that depend on both the number of tasks, and the amount of inequality in the t^z's, however, *can* give an interior solution to the problem, while remaining intuitively appealing.

[8] $$\max_{\{t^z\}_z}\left\{\sum_Z\left[E(h,t^z - f(t^z))f(t^z)\right] - C(\sum_Z (t^z)^2, \sum_Z \lambda(t^z > 0))\right\}$$

subject to $\sum_Z t^z = T$

Lemma. Workers split their time equally between all tasks that they perform in positive amounts.

Proof. The first order condition to the maximization problem in equation [8] is:

$$\begin{cases} E_t f(1-f_t) - Ef_t - 2C_{HHI}t^z = \delta & if\ t^z > 0 \\ E_t f(1-f_t) - Ef_t - C_N(HHI,N) < \delta & if\ t^z = 0 \end{cases}$$

where δ is a constant Lagrangian multiplier. Then if $t^{z_1} > 0$ and $t^{z_2} > 0$ for some z_1 and z_2,it must be $t^{z_1} = t^{z_2} = t^{z*}$. This follows from the fact that all tasks are equivalent, and hence the function E is independent of z. **QED**.

Since a tiny amount of knowledge in each task eliminates a discrete cost, workers in the model will always be "generalists", in the sense that they choose positive values of t^z for all z. However, they will generally set $t^z = \varepsilon$ (some arbitrarily small value) for some of the tasks. For the purposes of the analysis below, assume that ε can be set close enough to zero that it can be ignored. If so, then the number of different tasks chosen by a worker can be usefully denoted $N = \frac{T}{t^*}$, where t* is the (equal) amount of time spent on each task performed by the worker in non-trivial amounts.

The maximization problem in equation [8] can then be rewritten as

[9] $$\max_N\left\{NE(h,\tfrac{T}{N} - f(\tfrac{T}{N}))f(\tfrac{T}{N}) - C(\tfrac{T^2}{N}, N)\right\}$$

Proposition 2. Only if ability and specialized human capital are significantly substitutable will high ability workers specialize less relative to low ability workers.

Proof. The first order condition for the maximization problem in equation [9] is

$$Ef + \tfrac{T}{N} E_t f(f_t - 1) - \tfrac{T}{N} Ef_t = C_N - C_{HHI} \tfrac{T^2}{N^2}$$

From this, we can derive N_h by taking the derivative of the first order condition with respect to h. Doing so gives the solution:

$$N_h = \frac{E_h(f_t \tfrac{T}{N} - f) - \tfrac{T}{N} E_{th} f(f_t - 1)}{SOC}$$

where SOC is the second order condition from the maximization problem. Hence, N_h is negative if and only if

$$E_h(f_t \tfrac{T}{N} - f) - \tfrac{T}{N} E_{th} f(f_t - 1) > 0 .$$

Using the fact f_t is between zero and one, we see that the more positive is E_{th} i.e., the more complementary are general and specialized human capital in production, the more positive the expression becomes[11]. Only if E_{th} is sufficiently negative will N_h be positive. This corresponds with the case where general and specialized human capital are substitutable. **QED**.

III. Conclusion

Proposition 2 shows that the correlation between ability, *h*, and the number of tasks performed, *N*, depends critically on the sign of the cross-partial derivative of productivity with respect to ability and specialized knowledge, E_{th}. When ability is substitutable in production for specialized knowledge, this term is negative, and workers with high ability benefit marginally less from specialized knowledge than low-ability workers. In such a case, high ability workers can benefit from applying themselves in a variety of different tasks, substituting their ability for

[11] In particular, if f is convex, then $E_{th} > 0$ implies the result. The convexity of f turns on the sign of E_{ttt}.

specialized knowledge in each. The sign of E_{th} in any particular production process remains as an important empirical question.

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